647510 **MSC2MatC202x** Seat No:_________

M.Sc. (Maths) Semester - 2 (*CBCS*) Examination
May/June -2021 [NEW COURSE]
Complex Analysis - 2(CORE)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figure to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1 (A) Find the linear fractional transformation maps $z_1 = -1, z_2 = i, z_3 = 1$ onto $w_1 = 0, w_2 = i, w_3 = \infty$ respectively. [4]

(B) Attempt any two. [10]

1. If $x_n = cis\frac{\pi}{2^n}$, then show that $x_1.x_2.x_3 \dots x_\infty = -1$.
2. Prove: $(\text{cis}\,\theta)^n = \text{cis}\,n\theta$, for $n \in N$.
3. Prove that $\tan\left[i\ln\left(\frac{x+iy}{x-iy}\right)\right] = \frac{2xy}{y^2-x^2}$.

Q.2 (A) Evaluate $\oint_c \frac{dz}{z-2}$ around circle $|z-2| = 4$. [4]

(B) Attempt any two. [10]

1. State and prove Cauchy's integral formula.
2. Using Cauchy's integral formula, evaluate $\oint_c \frac{e^z}{z(1-z)^3}\,dz$ where$C$: $|z| = \frac{1}{2}$.
3. Evaluate $\oint_c |z|^2\,dz$ around the square with vertices $(0,0), (1,0), (1,1)$ and $(0,1)$.

Q.3 (A) State and prove Fundamental theorem of Algebra. [4]

(B) Attempt any two. [10]

1. State and prove Liouville's theorem.
2. Evaluate $\oint_C \frac{5z+7}{z^2+2z-3}dz$, where C is circle $|z-2| = 2$.
3. State Identity theorem.

Q.4 (A) State and prove argument principle theorem. [4]

(B) Attempt any two. [10]

1. Write statement of Schwarz's lemma and give necessary explaination.
2. State and prove open mapping theorem.
3. Suppose that f and g are analytic in an open set containing a circle C and its interior. If $|f(z)| > |g(z)|, \forall\, z \in C,$ then show that f and $f + g$ have the same number of zeros inside the circle C.

Q.5 (A) Using residue theorem Evaluate $\oint_c e^z \sec\pi z\; dz$, where C is the unit circle $|z| = 1$. [4]

(B) Attempt any two. [10]

1. State and prove Residue theorem.
2. Determine the residue of $f(z) = \frac{3z-4}{z(z-1)(z-2)}$ of each of its poles in the finite z – plane.
3. Find the zero's of $(z-1)^3$.

****